DOUBLING OF THE OSCILLATION PERIOD WITH C-BIFURCATIONS IN PIECEWISE-CONTINUOUS SYSTEMS

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The behavior of a dynamic system is considered in the event of violation of the conditions of existence of a periodic state due to alteration of the sequence of passage of the phase trajectory through the piecewise continuity domains. It is shown that doubling of the oscillation period is possible under these conditions.

Piecewise-continuous dynamic systems afford a good description of the behavior of a broad class of machines (relay, vibroimpact, dry friction machines, etc.). Such systems entail the possibility of specific violations of the conditions of existence of periodic motion due to changes in the number of segments of the phase trajectories of which the trajectory of this motion is "matched" (C-bifurcations). In [1] it was shown that C-bifurcation makes possible not only transition of one type of periodic state into a state of another type but also the merging of these two states and their subsequent disappearance. (However, no examples of dynamic systems in which this possibility was realized were cited). The present paper demonstrates the possibility of yet another mode of system behavior with C-bifurcations, namely the doubling of the oscillation period. The period doubling criterion, as well as the criterion of merging and disappearance of two states, are obtained. It is shown that of the three possible cases of behavior of a system subject to C-bifurcations, only one case, namely transition of a periodic state of one type into a state of another type, is a consequence of the choice of mathematical model of the system in question, i.e. is noncoarse with respect to the class of nonlinear characteristics [2]. The first examples concern the "hard" doubling of the oscillation period and the case of merging of two states with C-bifurcations in a linear system with a displacement limiter. The results are then used to isolate the approximate parameter domains which allow subharmonic oscillations of order $1/2$ in a system with a symmetric nonlinearity characteristic in the form of a third-degree polynomial. The isolated domains are in good agreement with the familiar results of experimental studies and simulation on an analog computer carried out by Hayashi [3].

1. Let us consider the dependence of the periodic motions of a piecewise-continuous dynamic system on a certain parameter. The phase space of such a system is divided by certain surfaces (with given conditions of matching of the phase trajectories) into domains in each of which the motions are described by differential equations with continuous and sufficiently smooth right sides.

As we know, various types of periodic motions are possible in piecewise-continuous systems. A periodic state of a given type is characterized by a completely defined sequence of passage of the phase point, and therefore of the phase trajectory matched in a certain way out of individual segments, through the piecewise-continuity domains. In the case of C-bifurcation the periodic motion trajectory passes through the boundary of one of the matching surfaces, which results in violation of the conditions of existence of this motion and corresponds to the appearance or disappearance of a segment of the trajectory in one of the piecewise continuity domains.

We shall show that C-bifurcation can result in sprouting of a "two-turn" state with a double oscillation period. Each of the turns of the trajectory of this motion corresponds to one of the types of periodic motion involved in the bifurcation.

The mathematical problem corresponding to the task we have posed can be formulated in a form similar to that of [1].

Let us consider the point map of some sufficiently smooth surface D which is dependent on the parameter μ. Let the map have a fixed point M° for $\mu = 0$. The phase trajectory L_0 of the corresponding periodic state which passes through the point M° also passes through the boundary Γ of the matching surface Π (Fig. 1). Let us isolate on the surface D the "line" S passing through M° which is mapped into the boundary Γ by phase trajectories close to L_0. The curve S separates the surface D into the two half-neighborhoods D_+ and D_-; the phase trajectories (for example, L_+ and L_-) emerging from these half-neighborhoods correspond to different equations of motion. We assume that the point mapping T is continuous in the neighborhood M°, and that its dependence on the phase coordinates and on the parameter in each of the half-neighborhoods is sufficiently smooth.

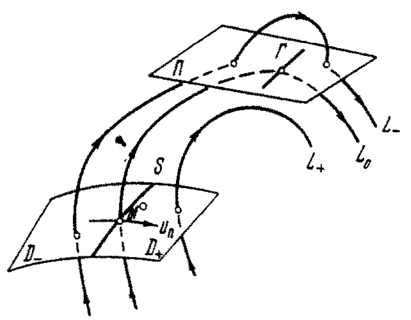

Fig. 1

Let us transfer the origin of the fixed point M° of the map. We choose one of the coordinate axes, e. g. u_n, in such a way that the domains D_+ and D_- are associated with differing signs of u_n. We can now write the equations of the mapping T linearized in the neighborhood of L_0 as follows. The mapping T^+,

$$u_k' = \sum_{s=1}^{n-1} a_{ks}u_s + a_{kn}^{+}u_n + b_k\mu + \dots \qquad (u_n \geqslant 0) \tag{1.1}$$

The mapping T^-,

$$u_k' = \sum_{s=1}^{n-1} a_{ks}u_s + a_{kn}^{-}u_n + b_k\mu + \dots \qquad (u_n \leqslant 0) \tag{1.2}$$

$$k = 1, 2, \dots, n$$

The two-turn periodic state is associated with a pair of fixed points M^* and M^{**} of the mapping T^+T^-. The coordinates of the fixed points can be found from the system of $2n$ equations

$$u_k^{**} = \sum_{s=1}^{n-1} a_{ks}u_s^* + a_{kn}^{+} u_n^* + b_k\mu + \dots \tag{1.3}$$

$$u_k^* = \sum_{s=1}^{n-1} a_{ks}u_s^{**} + a_{kn}^{-}u_n^{**} + b_k\mu + \dots \quad (k = 1, 2, \dots, n)$$

$$u_n^* > 0, \qquad u_n^{**} < 0 \tag{1.4}$$

We introduce the differences

$$\delta_k = u_k^{**} - u_k^* \qquad (k = 1, 2, \dots, n)$$

Now, instead of Eqs. (1.3) we obtain the linear approximations

$$u_k^{**} = \sum_{s=1}^{n-1} a_{ks} u_s^* + a_{kn}^+ u_n^* + b_k \mu \tag{1.5}$$

$$\delta_k + \sum_{s=1}^{n-1} a_{ks} \delta_s + a_{kn}^+ \delta_n = u_n^{**} (a_{kn}^+ - a_{kn}^-) \tag{1.6}$$

From system (1.6) we obtain

$$\delta_k = \frac{\beta_k}{\chi^+(-1)} u_n^{**} \qquad (k = 1, 2, \ldots, n) \tag{1.7}$$

Here $\chi^+(-1)$ denotes the characteristic polynomial $\chi^+(z)$ of the mapping T^+ for $z = -1$, and β_k denotes some functions of the coefficient a_{ks}. We assume that the eigenvalues of the characteristic matrix differ from unity.

The difference δ_n is given by

$$\delta_n = u_n^{**} - u_n^* = \frac{\chi^+(-1) - \chi^-(-1)}{\chi^+(-1)} u_n^{**}$$

Hence,

$$u_n^* \chi^+(-1) = u_n^{**} \chi^-(-1) \tag{1.8}$$

The dependence of the required coordinates on the parameter μ can be obtained by substituting the values

$$u_k^{**} = u_k^* + \frac{\beta_k}{\chi^-(-1)} u_n^*$$

in accordance with (1.7) and (1.8) into Eqs. (1.5),

$$u_k^* = \sum_{s=1}^{n-1} a_{ks} u_s^* + \left[a_{kn}^+ - \frac{\beta_k}{\chi^-(-1)} \right] u_n^* + b_k \mu \tag{1.9}$$

If the eigenvalues of the matrix consisting of the coefficients of the required coordinates of Eqs. (1.9) are different from unity, then Eqs. (1.9) have a nonzero solution. However, by virtue of the piecewise continuity of the system under consideration we must also require fulfillment of conditions (1.4). In accordance with (1.9) and (1.8) the values u_n^* and u_n^{**} can be expressed as

$$u_n^* = A_n \chi^-(-1) \mu, \qquad u_n^{**} = A_n \chi^+(-1) \mu \tag{1.10}$$

From (1.10) we infer that inequalities (1.4) are fulfilled if

$$\chi^+(-1)\, \chi^-(-1) < 0 \tag{1.11}$$

Condition (1.11) is the criterion of doubling of the oscillation period with C-bifurcations. It is practically convenient, since to solve the problem of appearance of a subharmonic oscillation we need merely determine the signs of the corresponding characteristic polynomials for $z = -1$ in the limiting case of coincidence of two periodic states. This does not entail a special choice of the system of phase coordinates as required in the theoretical proof of the possibility of period doubling. Condition (1.11) is related to the character of stability of periodic states and is invariant relative to the choice of coordinates.

2. One of the important problems of the theory of bifurcations of periodic motions is the determination of the so-called "dangerous boundaries". The notion of dangerous boundaries introduced in [4] for dynamic systems with analytic right sides can be naturally extended to the case of C-bifurcations in piecewise-continuous systems. Following Andronov (see the Preface to [4]), we define a dangerous C-bifurcation boundary as the boundary of existence of a periodic state whose slightest violation results in uncontrollable (by choice of a sufficiently small disturbance) growth of the deviation of the state of motion from the state under consideration.

The case of merging of two periodic states followed by their disappearance is one of dangerous C-bifurcation. Linearized equations (1.1), (1.2) readily yield the dangerous C-bifurcation criterion in a form similar to condition (1.11).

Let a fixed point M^* of the mapping T^+ and a fixed point M^{**} of the mapping T^- exist for some value of the parameter μ. Setting the values $u_k' = u_k = u_k^*$ in (1.1) and the values $u_k' = u_k = u_k^{**}$ in (1.2), we obtain the following values for the coordinates:

$$u_n^* = \frac{B_n}{\chi^+(+1)}\mu + \ldots, \qquad u_n^{**} = \frac{B_n}{\chi^-(+1)}\mu + \ldots \tag{2.1}$$

Requiring fulfillment of the conditions $u_n^* > 0$ and $u_n^{**} < 0$, we arrive at the following dangerous C-bifurcation criterion:

$$\chi^+(+1)\chi^-(+1) < 0 \tag{2.2}$$

Note 2.1. We know [5] that doubling of the oscillation period and merging of two periodic states with their subsequent disappearance can occur in dynamic systems with analytic nonlinearity characteristics on the boundary of a steady state. The former takes place when one of the roots of the characteristic polynomial becomes -1 on the boundary N_-,

$$\chi(-1) = 0 \tag{2.3}$$

the latter occurs for $z = +1$ on the boundary N_+,

$$\chi(+1) = 0 \tag{2.4}$$

With the corresponding transition from piecewise-continuous characteristics to continuous characteristics, conditions (1.11) and (2.2) become Eqs. (2.3) and (2.4), so that the general picture of the dependence of trajectory behavior on the parameter remains unchanged.

This fact characterizes the coarseness of the parameter space of a dynamic system relative to the class of nonlinear characteristics [2] and points to a profound connection between the behavior of a real dynamic system and the C-bifurcations of motion in the chosen piecewise-continuous mathematical model of this system; of the three possible cases of behavior of a system with C-bifurcations only the case of transition of a periodic state of one type into a state of another type is a consequence of the choice of model. It is therefore expedient to study the C-bifurcations of the periodic motions of piecewise-linear models of dynamic systems in investigating the qualitative structure of the parameter space; this is because such models are more amenable to analysis than the bifurcations N_- and N_+ in models with continuous nonlinearity characteristics. An example using this approach appears in the last section of the present paper.

Note 2.2. The problem of preservation or violating of stability of a state with C-bifurcation can be solved by analyzing the roots of the corresponding characteristic polynomials. However, certain general conclusions can be drawn for specific cases.

Let both periodic states taking part in a bifurcation be stable. Then doubling of the period and merging of two states are impossible, since the signs of the characteristic polynomials for $z = -1$ or $z = +1$ are the same, so that conditions (1.11) and (2.2) are not satisfied. This case of C-bifurcation corresponds to a change in the type of periodic motion.

Suppose now that a C-bifurcation involves transition of a stable state into an unstable one. The continuity of the change in topology of the phase space is preserved [5, 6] if the transition is accompanied either by the appearance of a stable oscillation or by the disappearance of an unstable oscillation of doubled period. We note that the latter case corresponds to a dangerous C-bifurcation.

3. For example, let us consider doubling of the period and dangerous C-bifurcations of forces oscillations of a linear oscillatory system on encountering a dispacement limiter (Fig. 2).

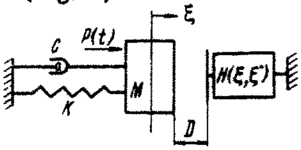

Fig. 2

The equation of system motion for $x < d$ in dimensionless form is

$$x^{\cdot\cdot} + 2\lambda x^{\cdot} + x = P(\tau) \tag{3.1}$$

where $P(\tau)$ is a T-periodic function of time, and the coefficient λ characterizes linear friction $(0 < \lambda < 1)$.

The solution of linear equation (3.1) can be written as

$$x(\tau) = p(\tau) + e^{-\lambda(\tau-\tau_0)}\{C_1 \sin\delta(\tau-\tau_0) + C_2 \cos\delta(\tau-\tau_0)\} \tag{3.2}$$

Here $p(\tau)$ is the particular solution of the inhomogeneous equation, i.e. the steady forced oscillations of the system; C_1 and C_2 are integration constants which are determined by the values of x_0 and $x^{\cdot}_0$ at the instant $\tau = \tau_0$; the coefficient $\delta = \sqrt{1-\lambda^2}$.

Let us rewrite solution (3.2) in the form of the equations of the point mapping corresponding to the segment of the phase trajectory lying between the point $M_i\{x_i, x^{\cdot}_i, \tau_i\}$ and the point $M_j\{x_j, x^{\cdot}_j, \tau_j\}$,

$$f_1(x_i^{\cdot}, \tau_i, \tau_j) = p_j - x_j + e^{-\lambda\tau_{ij}}\left\{(x_i - p_i)\left(\frac{\lambda}{\delta}\sin\delta\tau_{ij} + \cos\delta\tau_{ij}\right) + \frac{x_i^{\cdot} - p_i^{\cdot}}{\delta}\sin\delta\tau_{ij}\right\} = 0 \tag{3.3}$$

$$x_j^{\cdot} = g(x_i^{\cdot}, \tau_i, \tau_j) = p_j^{\cdot} + e^{-\lambda\tau_{ij}} \times \left\{(x_i^{\cdot} - p_i^{\cdot})\left(\cos\delta\tau_{ij} - \frac{\lambda}{\delta}\sin\delta\tau_{ij}\right) - \frac{x_i - p_i}{\delta}\sin\delta\tau_{ij}\right\} \quad \tau_{ij} = \tau_j - \tau_i$$

If the system oscillations do not reach the limiter, then Eqs. (3.3) define the behavior of the system both in the transient state (trajectory M_0M_1 in Fig. 3a) and in the steady state, where

$$x(\tau) = p(\tau), \qquad x^{\cdot}(\tau) = p^{\cdot}(\tau) \tag{3.4}$$

Equations (3.3) are necessary in this case for writing out the characteristic polynomial $\chi^+(z) = 0$. The points $M_0 = M_i$ and $M_1 = M_j$ correspond to $x_i = x_j = 0$. Varying Eq. (3.3) in the variables $x_i^{\cdot}$, τ_i, $x_j^{\cdot}$, τ_j in the neighborhood of periodic

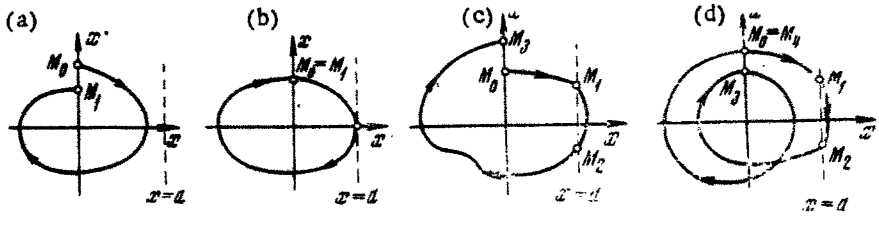

Fig. 3

motion (3.4) and setting $\tau_{ij} = T$, $\delta x_j^{\cdot} = z\delta x_i^{\cdot}$, $\delta\tau_j = z\delta\tau_i$, we obtain

$$\chi^+(z) = \begin{vmatrix} (\partial f / \partial x_i^{\cdot})_{01} & (\partial f / \partial \tau_i)_{01} + z\,(\partial f / \partial \tau_j)_{01} \\ (\partial g / \partial x_i^{\cdot})_{01} - z & (\partial g / \partial \tau_i)_{01} + z\,(\partial g / \partial \tau_j)_{01} \end{vmatrix} \tag{3.5}$$

Substituting the values of the partial derivatives obtained from Eqs. (3.3) into (3.5), we arrive at the expression

$$\chi^+(z) = x_0^{\cdot}\,(z^2 - 2ze^{-\lambda T}\cos(\delta T) + e^{-2\lambda T}) \tag{3.6}$$

From this we obtain the value of the characteristic polynomial for $z = \pm 1$,

$$\chi^+(\pm 1) = x_0^{\cdot}\,[(e^{-\lambda T} \mp \cos\delta T)^2 + \sin^2\delta T] > 0 \tag{3.7}$$

If the system oscillations reach the limiter, then Eqs. (3.3) determine its behavior only on the segments M_0M_1 and M_2M_3 of the phase trajectory (Fig. 3c). Let the equations of the point mapping of the surface $x = d$ into itself corresponding to the segment M_1M_2 of the trajectory of motion with an operating limiter be expressed in the form

$$\tau_2 - \tau_1 - x_1^{\cdot}\,\Phi(\tau_1) = 0, \qquad x_2^{\cdot} = x_1^{\cdot}\,\Phi_1(\tau_1) \tag{3.8}$$

Varying Eqs. (3.3) and (3.8), respectively, for the trajectory segments M_0M_1, M_1M_2, M_2M_3 in the neighborhood of forced oscillation (3.4), setting

$$\tau_3 - \tau_0 = T, \quad \delta x_3^{\cdot} = z\delta x_0^{\cdot}, \quad \delta\tau_3 = z\delta\tau_0$$

and carrying out the necessary operations, we arrive at the following characteristic polynomial:

$$\chi^-(z) = \delta^{-1} x_0^{\cdot}\, x_1^{\cdot\cdot}\, z e^{-\lambda T}\,(1 + x_1^{\cdot\cdot}\,\Phi - \Phi_1)\sin\delta T \tag{3.9}$$

In the case of C-bifurcation (Fig. 3b) the value $x_0^{\cdot} > 0$, and $x_1^{\cdot\cdot} < 0$, since the point of tangency corresponds to $x_{\max}$. Hence,

$$\operatorname{sign}\chi^-(z) = \operatorname{sign}\{z\,(1 + x_1^{\cdot\cdot}\,\Phi - \Phi_1)\sin\delta T\} \tag{3.10}$$

By virtue of (3.7) the value $\chi^+(\pm 1) > 0$, so that in the problem under consideration we obtain the condition of a dangerous C-bifurcation (2.2) involving the merging of two states followed by their disappearance in the form

$$(1 + x_1^{\cdot\cdot}\,\Phi - \Phi_1)\sin\delta T < 0 \tag{3.11}$$

and period doubling condition (1.11) in the form

$$(1 + x_1^{\cdot\cdot}\,\Phi - \Phi_1)\sin\delta T > 0 \tag{3.12}$$

The first factor in (3.11), (3.12) reflects the properties of the limiter; the second factor reflects the properties of the linear oscillatory system.

Let us note an important distinctive feature of the bifurcation in question. Since $(\partial f / \partial \tau_j)_{01} = x_1^{\cdot} \to 0$ at the instant of contact, it follows that the polynomial $\chi^-(z)$ has the roots $z_1 \to 0$ and $z_2 \to \infty$. which indicates the instability of the nonlinear state taking part in the bifurcation. Hence, on fulfillment of (3.11) the state of stable forced oscillations of the linear system merges with the unstable nonlinear state of motion. If condition (3.12) is fulfilled, then the stable state of the linear system becomes an unstable nonlinear state. This is accompanied either by the vanishing of the unstable state or by the appearance of a stable subharmonic state of doubled period.

On merging of the stable periodic state with the unstable state the dynamic system experiences a "hard" transition to another stable state of operation. In the simplest case the latter state takes the form of a periodic motion of the same type as the vanished unstable state. Condition (3.11) becomes condition (3.12) for $\sin\delta T = 0$ or

$$T\sqrt{1-\lambda^2} = n\pi \qquad (n = 0, 1, \ldots) \tag{3.13}$$

Expression (3.13) defines certain nodal bifurcation points in the parameter space of the dynamic system from which several bifurcation boundaries of various types must emerge.

The results obtained in the present study and the behavior of a system on the stability boundary of a state as known to us from bifurcation theory [5] enable us to "synthesize" the possible structures of these nodes qualitatively. Thus, Fig. 4 shows the two simplest structures. Here the boundaries N_+ correspond to the merging of stable and unstable states of a single type when the root of the characteristic polynomial assumes the value +1. The boundary N_- corresponds to soft doubling of the period on appearance of the root $z = -1$. On the boundary C_1 the state involving two segments of limiter operation into a one-segment state with preservation of the oscillation period and stability.

The state of stable forced oscillations of a linear system changes into a nonlinear state occasioned by the limiter at the boundaries C_2 and C_3. This can be accompanied either by a soft (Fig. 4a) or by a hard (Fig. 4b) doubling of the oscillation period at the C-bifurcation boundary C_3. To determine which of the cases of doubling corresponds to the forced oscillations under consideration, we must investigate the stability of the two-turn periodic state with C-bifurcation. The trajectory of this state of motion (Fig. 3d) consists of the segments M_0M_1, M_2M_3 and M_3M_4 defined by Eqs. (3.3) and of the segment M_1M_2 defined by (3.8). After some simple but cumbersome operations of deriving and simplifying the appropriate characteristic polynomial, we arrive at the following expression:

$$\chi(z) = -\delta^{-1} z x_1^{\cdot\cdot} x_0^{\cdot 2} e^{-2\lambda T}(1 + x_1^{\cdot\cdot}\Phi - \Phi_1)\sin 2\delta T \tag{3.14}$$

Thus, one of the roots of $\chi(z) = 0$ also increases without limit in the case of a doubled oscillation period as $x_1^{\cdot} \to 0$, and the state turns out to be unstable. Hence, in the domains

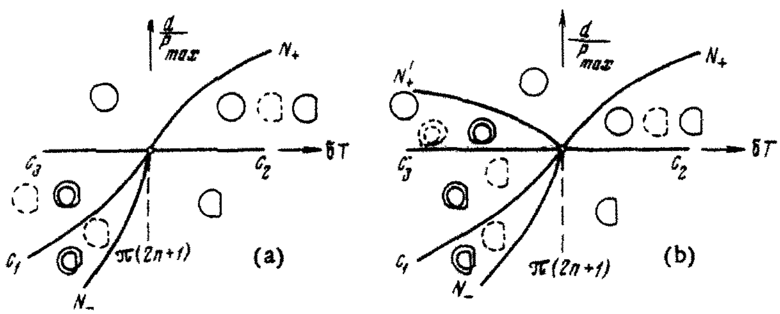

Fig. 4

of parameter variation satisfying condition (3.12) we expect a hard doubling of the oscillation period (Fig. 4b). Quite naturally, a structure of decomposition of the parameter space into domains of periodic motions of various types more complicated than that in Fig. 4 is possible in the neighborhood of bifurcation nodes.

4. The above condition of period doubling with C-bifurcations (expression (3.12)) is quite coarse relative to the nonlinearity characteristic of the oscillatory system under consideration. It is therefore interesting to verify whether the conditions obtained for a piecewise-linear system are also applicable to systems with a "smooth" nonlinearity characteristic.

To this end we begin by considering linear oscillatory system (3.1) with two symmetrically positioned fixed limiters. As our relation (3.8) between the coordinates of the initial and final points of thr trajectory M_1M_2 in this case we can take the law of instantaneous impact with the velocity restitution coefficient R $(0 < R < 1)$,

$$\tau_2 - \tau_1 = 0, \qquad x_2^{\cdot} = -Rx_1^{\cdot} \tag{4.1}$$

Hence, in expression (3.8) we have

$$\Phi(\tau_1) = 0, \qquad \Phi_1(\tau_1) = -R$$

The characteristic polynomials corresponding to symmetric periodic states are reducible to the form

$$\chi^+(z) = x_0^{\cdot}(z^2 + 2ze^{-1/_2\lambda T}\cos{}^1/_2\delta T + e^{-\lambda T}) \tag{4.2}$$
$$\chi^-(z) = \delta^{-1}z(1+R)\,x_1^{\cdot\cdot}\,e^{-1/_2\lambda T}\sin{}^1/_2\delta T$$

and period doubling condition (1.11) reduces to $\sin{}^1/_2\delta T < 0$. We find that the sprouting of subharmonic oscillations of order $^1/_2$ is possible if the ratio of the natural frequency δ to the frequency $\omega = 2\pi / T$ of external excitation satisfies one of the conditions

$$2n + 1 < \delta / \omega < 2(n + 1), \qquad n = 0, 1, 2, \ldots \tag{4.3}$$

Let us consider the system with the nonlinearity characteristic expressed by the third-degree polynomial [3]

$$x^{\cdot\cdot} + \lambda x^{\cdot} + c_1 x + c_3 x^3 = B\cos 2\tau \tag{4.4}$$
$$(\lambda \ll 1,\ c_1 + c_3 = 1)$$

We obtain an approximate relationship between the natural oscillation frequency and the amplitude from (4.4) by setting $B = 0$ and seeking the solution in the form $x = X\cos\delta\tau$.

$$\delta^2 \approx c_1 + {}^3/_4\, c_3 X^2 \tag{4.5}$$

The amplitude of the forced oscillations can be determined from the external force frequency by means of the approximate expression [3]

$$X \approx \frac{B}{|1-\omega^2|} \tag{4.6}$$

Let us use expression (4.3) obtained for a piecewise-linear system to estimate the domains of appearance of subharmonic oscillations of order $^1/_2$ in system (4.4). To this end we substitute the value of δ obtained from (4.5), (4.6) into condition (4.3),

$$2n + 1 < \frac{1}{\omega}\left[c_1 + \frac{3c_3B^2}{4(1-\omega^2)^2}\right]^{1/2} < 2(n+1) \quad (n = 0, 1, 2, \ldots) \tag{4.7}$$

The above result can be compared with the data obtained both experimentally and by analog computer by Hayashi for subharmonic oscillations of order $^1/_2$ (see [3]). Setting $\omega = 2$, $c_1 \approx 0$ and taking as the unit the value of B^* corresponding to the center of the first domain, we obtain from (4.7) the following intervals of relative amplitude B/B^* in which the subharmonic oscillations under consideration are possible:

$$(0.66 - 1.33),\ (2.00 - 2.66),\ (3.33 - 4.00), \ldots$$

Hayashi [3] obtained only two intervals for the case of a zero constant external force component (Figs. 7.22 and 7.25 in [3]).

After a similar division by B^* these intervals turn out to be as follows: (0.9–1.1), (2.0–2.3) according to the results obtained by analog computer, and (0.8–1.2), (1.8–2.1) according to the experimental results. A damped system was studied in both cases.

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QUALITATIVE INVESTIGATION OF THE STRESS-STRAIN STATE OF A SANDWICH PLATE

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The problem of the passage to the limit from three-dimensional problems of elasticity theory to two-dimensional problems has been investigated in [1, 2] for multilayered plates. A first iterarion process has been constructed therein on the basis of methods developed in [3, 4].

A construction of homogeneous solutions of elasticity theory problems for sandwich plates of symmetric configuration is given below. As in the case of a homogeneous plate [6], it is shown that the complete solution consists of a biharmonic, potential and vortex solution. The potential and vortex solutions are in the nature of an edge effect. Comparing them to the case of a homogeneous plate, shows that the edge effects can be both weaker and stronger, depending on the physical and geometric parameters of the sandwich plate.

The accuracy of some applied theories [6] is analyzed on the basis of the solution constructed, and limits for their applicability are established.

1. Let us consider a sandwich plate comprised of isotropic layers which are symmetric relative to the middle plane of the middle layer (Fig. 1). Let μ_i denote the shear modulus, i the number of the layer, σ_i the Poisson's ratio. Let the outer layers of thickness δ have the elastic characteristics ν_1 and μ_1, and the inner layer of thickness $2h$ the elastic characteristics ν_2 and μ_2.

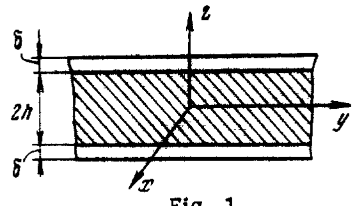

Fig. 1

Let us assume the outer plane faces to be stress-